Analysis of 2^n factorial experiments with exponentially distributed response variable: A comparative study

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ABSTRACT

The exponential distribution is an important distribution in the analysis of lifetime data. In many industrial designed experiments, the response variable is a lifetime random variable, for example, lifetime of produced items. This paper assesses and compares the performance of generalized linear models (GLM) with respect to log transformation (LOG) approach for saturated and unsaturated factorial experiments. The comparison was mainly simulation based. The performance evaluation was based on the coverage and expected length of confidence intervals (CI) for the expected responses for saturated as well as unsaturated factorial experiments at the prescribed confidence coefficient. The coverage results of log-transformation approach were uniformly superior to GLM for two replications (very small sample) which was most frequently encountered situation in the industrial experiments.

The most widely used technique for the analysis of designed experiments is ANOVA technique. The optimality properties of ANOVA based F-test heavily depend on normality and homogeneity assumptions. Certainly there are many situations, where the non-normal response and nonhomogeneous variance is a fact. Lewis *et al*. (2001a) gave many examples of designed experiments with non-normal responses. These responses may be related non-linearly with the input factors and the variance is not constant but is a function of mean. Moreover, only very small number of replications is available due to stringent practical constraints. Traditionally in such situations, a suitably transformed response variable (most frequently a variance stabilizing transformation) is analyzed using ANOVA based F-test and the results are transformed back to the original scale. But this approach has many drawbacks, Myers and Montgomery (1997). Several authors have used GLM, introduced by Nelder and Wedderburn (1972) as a powerful alternative to data transformation. Myers and Montgomery (1997) gave a tutorial on GLM and Lewis

et al. (2001b) attempted a simulation study on analysis of designed experiments involving non-normal response variables using GLM based on a pre-specified parameter combination each from the Binomial, Poisson and Gamma distributions and advocated the use of GLM for small samples.

Patil and Kulkarni (2011) attempted an in-depth comparative study among the traditional approaches (log-transformation (LOG) and ANOVA) and GLM for saturated 2-level factorial experiments with exponentially distributed response variables. The comparison was based on coverage results and expected length of confidence intervals (E(LOCI)) for the expected responses. They observed that the LOG approach outperforms GLM approach for two replications which is most typically encountered situation in industrial experiments but the coverages are not up to the desired level. They suggested a modification to GLM approach to improve its performance for interval estimation of the expected response and observed that the modified version of GLM performed uniformly best than the LOG approach. They also focused on the power functions of the tests of hypothesis for testing significance of underlying factorial effects. t-test is used for testing significance of the regression coefficients under LOG and ANOVA approaches. For GLM two types of tests are employed: t-test and deviance test. They observed that t-test based

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